OPERATIONAL RESEARCH II

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MARKOV CHAIN

Tugas: Latihan Soal

1 In Smalltown, 90% of all sunny days are followed by sunny days, and 80% of all cloudy days are followed by cloudy days. Use this information to model Smalltown's weather as a Markov chain.

2 Consider an inventory system in which the sequence of events during each period is as follows. (1) We observe the inventory level (call it i) at the beginning of the period. (2) If $i \leq 1$, $4 - i$ units are ordered. If $i \geq 2$, 0 units are ordered. Delivery of all ordered units is immediate. (3) With probability $\frac{1}{3}$, 0 units are demanded during the period; with probability $\frac{1}{3}$, 1 unit is demanded during the period; and with probability $\frac{1}{3}$, 2 units are demanded during the period. (4) We observe the inventory level at the beginning of the next period.

Define a period's state to be the period's beginning inventory level. Determine the transition matrix that could be used to model this inventory system as a Markov chain.

3 A company has two machines. During any day, each machine that is working at the beginning of the day has a $\frac{1}{3}$ chance of breaking down. If a machine breaks down during the day, it is sent to a repair facility and will be working two days after it breaks down. (Thus, if a machine breaks down during day 3, it will be working at the beginning of day 5.) Letting the state of the system be the number of machines working at the beginning of the day, formulate a transition probability matrix for this situation.

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16.2-1. Assume that the probability of rain tomorrow is 0.5 if it is raining today, and assume that the probability of its being clear (no rain) tomorrow is 0.9 if it is clear today. Also assume that these probabilities do not change if information is also provided about the weather before today.

(a) Explain why the stated assumptions imply that the *Markovian property* holds for the evolution of the weather.

(b) Formulate the evolution of the weather as a Markov chain by defining its states and giving its (one-step) transition matrix.

16.3-1. Reconsider Prob. 16.2-1.

C **(a)** Use the routine *Chapman-Kolmogorov Equations* in your OR Courseware to find the n-step transition matrix $\mathbf{P}^{(n)}$ for $n = 2, 5, 10, 20$.

(b) The probability that it will rain today is 0.5. Use the results from part (*a*) to determine the probability that it will rain n days from now, for $n = 2, 5, 10, 20$.

C **(c)** Use the routine *Steady-State Probabilities* in your OR Courseware to determine the steady-state probabilities of the state of the weather. Describe how the probabilities in the n-step transition matrices obtained in part (*a*) compare to these steady-state probabilities as n grows large.