

## Flow Shop Scheduling

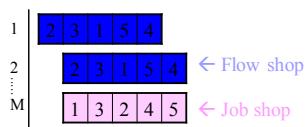
Mata Kuliah: Penjadwalan Produksi  
Teknik Industri – Universitas Brawijaya

### Definitions

- Contains  $m$  different machines.
- Each job consists  $m$  operators in different machine.
- The flow of work is unidirectional.
- Machines in a flow shop =  $1, 2, \dots, m$
- The operations of job  $i$ ,  $(i,1) (i,2) (i,3) \dots (i, m)$
- Not processed by machine  $k$ ,  $P(i, k) = 0$

### Flow Shop Scheduling

The processing sequence on each machine are all the same.

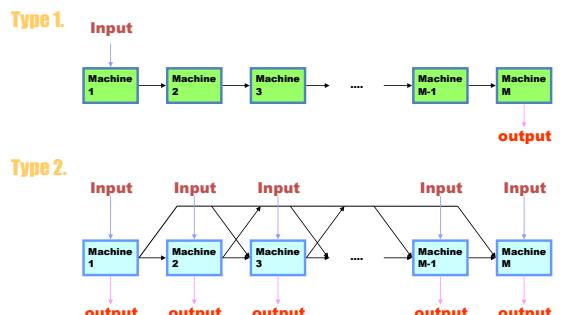


$n!$  - flow shop permutation schedule

$\underbrace{n! n! \dots n!}_{(n!)^m}$  - Job shop

$$\frac{(n!)^m}{k} \quad k : \text{constraint} \quad (\text{: routing problem})$$

### Workflow in a flow shop



## Johnson's Rule

Step1 : Find  $\min_i \{t_{ij_1}, t_{ij_2}\}$

Step2a : If the min  $t$  requires machine 1, place the job in the first available position in sequence. go to step3.

Step2b : If the min  $t$  requires machine 2, place the job in the first available position in sequence. go to step3.

Step3 : Remove the assigned job from consideration and return to step 1 until all positions in sequence are filled.

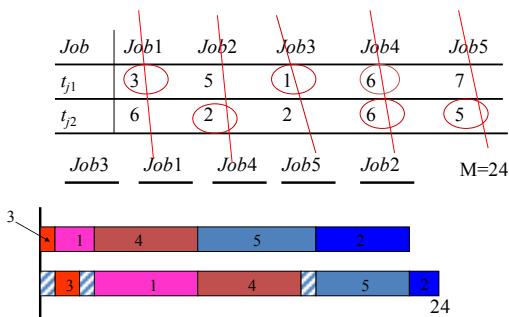
## Ex.

Stage	U	Min $t_{jk}$	Assignment	Partial Schedule
1	1,2,3,4,5	$t_{31}$	3=[1]	3 x x x x
2	1,2,4,5	$t_{22}$	2=[5]	3 x x x 2
3	1,4,5	$t_{11}$	1=[2]	3 1 x x 2
4	4,5	$t_{52}$	5=[4]	3 1 x 5 2
5	4	$t_{11}$	4=[3]	3 1 4 5 2

Note:

Johnson's rule can find an optimum with two machines  
Flow shop problem for makespan problem.

## Ex.



## The B&B for Makespan Problem

### The Ignall-Schrage Algorithm (Baker p.149)

- A lower bound on the makespan associated with any completion of the corresponding partial sequence  $\sigma$  is obtained by considering the work remaining on each machine. To illustrate the procedure for  $m=3$ .

For a given partial sequence  $\sigma$ , let

$q_1$ = the latest completion time on machine 1 among jobs in  $\sigma$ .

$q_2$ = the latest completion time on machine 2 among jobs in  $\sigma$ .

$q_3$ = the latest completion time on machine 3 among jobs in  $\sigma$ .

The amount of processing yet required of machine 1 is  $\sum_{j \in \sigma^c} t_{j1}$

## The Ignall-Schrage Algorithm

In the most favorable situation, the last job

- 1) Encounters no delay between the completion of one operation and the start of its direct successor, and
- 2) Has the minimal sum ( $t_{j2} + t_{j3}$ ) among jobs  $j$  belongs to  $\sigma'$

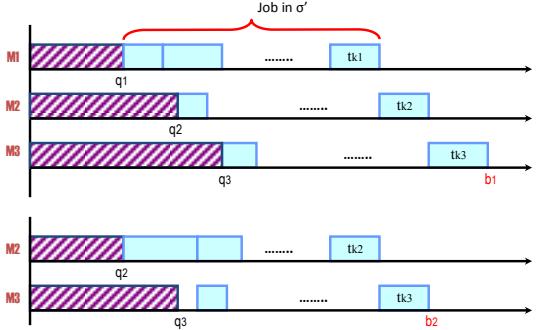
Hence one lower bound on the makespan is  $b_1 = q_1 + \sum_{j \in \sigma'} t_{j1} + \min_{j \in \sigma'} \{t_{j2} + t_{j3}\}$

A second lower bound on machine 2 is  $b_2 = q_2 + \sum_{j \in \sigma'} t_{j2} + \min_{j \in \sigma'} \{t_{j3}\}$

A lower bound on machine 3 is  $b_3 = q_3 + \sum_{j \in \sigma'} t_{j3}$

The lower bound proposed by Ignall and Schrage is  $B = \max\{b_1, b_2, b_3\}$

## The Ignall-Schrage Algorithm



## Ex. B&B

$m=3$

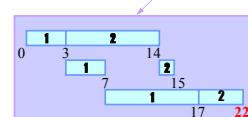
j	1	2	3	4
$t_{j1}$	3	11	7	10
$t_{j2}$	4	1	9	12
$t_{j3}$	10	5	13	2

For the first node:  $\sigma = 1$

$$\begin{aligned} q_1 &= t_{11} = 3 \\ q_2 &= t_{11} + t_{12} = 7 \\ q_3 &= t_{11} + t_{12} + t_{13} = 17 \\ \text{The lower bound} &= \\ b_1 &= 3 + 23 + 6 = 37 \\ b_2 &= 7 + 22 + 2 = 31 \\ b_3 &= 17 + 20 = 37 \\ B &= \max(37, 31, 37) = 37 \end{aligned}$$

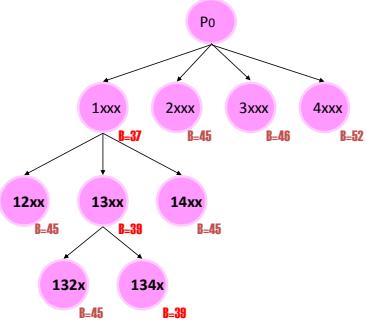
$$6 = \min \left\{ \begin{array}{l} 1+5 \\ 9+13 \\ 12+2 \end{array} \right\}$$

Partial Sequence	( $q_1, q_2, q_3$ )	( $b_1, b_2, b_3$ )	B
1xxx	(3, 7, 17)	(37, 31, 37)	37
2xxx	(11, 12, 17)	(45, 39, 42)	45
3xxx	(7, 16, 29)	(37, 35, 46)	46
4xxx	(10, 22, 24)	(37, 41, 52)	52
12xxx	(14, 15, 22)	(45, 38, 37)	45
13xx	(10, 19, 32)	(37, 34, 39)	39
14xx	(13, 25, 27)	(37, 40, 45)	45
132x	(21, 22, 37)	(45, 36, 39)	45
134x	(20, 32, 34)	(37, 38, 39)	39



$$\begin{aligned} b_1 &= q_1 + \sum_{j \in \sigma} t_{ji} + \min_{j \in \sigma} \{t_{j2} + t_{j3}\} \\ &= 14 + (7+10) + \min \left\{ \begin{array}{l} 9+13 \\ 12+2 \end{array} \right\} \\ &= 45 \end{aligned}$$

## Ex. B&B



## Hw.

Consider the following four-job three-machine problem

	1	2	3	4
t <sub>j1</sub>	13	7	26	2
t <sub>j2</sub>	3	12	9	6
t <sub>j3</sub>	12	16	7	1

- Find the min makespan using the basic Ignall-Schrage algorithm. Count the nodes generated by the branching process.
- Find the min makespan using the modified algorithm.

## Heuristic Approaches

Traditional B&B:

- The computational requirements will be severe for large problems
- Even for relatively small problems, there is no guarantee that the solution can be obtained quickly,

Heuristic Approaches

- can obtain solutions to large problems with limited computational effort.
- Computational requirements are predictable for problem of a given size.

## CDS (Campbel, Dudek and Smith)

Its strength lies in two properties:

- It use Johnson's rule in a heuristic fashion
- It generally creates several schedules from which a "best" schedule can be chosen.

The CDS algorithm corresponds to a multistage use if Johnson's rule applied to a new problem, derived from the original, with processing times  $t'_{jl}$  and  $t'_{j2}$ . At stage 1,  $t'_{jl} = t_{jl}$  and  $t'_{j2} = t_{jm}$

## CDS

In other words, Johnson's rule is applied to the first and  $m$ th operations and intermediate operations are ignored. At stage 2  
 $t_{j1}^* = t_{j1} + t_{j2}$  and  $t_{j2}^* = t_{jm} + t_{j,m-1}$

That is, Johnson's rule is applied to the sums of the first two and last two operation processing times. In general at stage  $i$ ,

$$t_{ji}^* = \sum_{k=1}^i t_{jk} \quad \text{and} \quad t_{j2}^* = \sum_{k=1}^i t_{j,m-k+1}$$

## CDS

**Step 1.** Set K=1. Hitung ( $m$ =jumlah mesin):

$$t_{i,1}^* = \sum_{k=1}^K t_{i,k} \quad \text{dan} \quad t_{i,2}^* = \sum_{k=1}^K t_{i,m-k+1}$$

**Step 2.** Gunakan Algoritma Johnson untuk penentuan urutan pekerjaan dengan menyatakan

$$t_{i,1} = t_{i,1}^* \quad \text{dan} \quad t_{i,2} = t_{i,2}^*$$

**Step 3.** Hitung makespan untuk urutan tersebut. Catat jadwal dan makespan yang dihasilkan

**Step 4.** Jika  $K=m-1$  maka pilih jadwal dengan makespan terpendek sebagai jadwal yang digunakan, lalu stop. Jika  $K < m-1$  maka  $K=K+1$  dan kembali ke Step 1.

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## CDS

Job i	Mesin 1	Mesin 2	Mesin 3
1	4	3	5
2	3	3	4
3	2	1	6
4	5	3	2
5	6	4	7
6	1	8	3

Job i	K=1		K=2	
	Mesin 1	Mesin 3	Mesin 1	Mesin 2
1	4	5	7	8
2	3	4	6	7
3	2	6	3	7
4	5	2	8	5
5	6	7	10	11
6	1	3	9	11

Set K=1

$$t_{1,1}^* = \sum_{k=1}^1 t_{1,k} = t_{1,1} = 4$$

$$t_{1,1}^* = \sum_{k=1}^2 t_{1,k} = t_{1,1} + t_{1,2} = 4 + 3 = 7$$

$$t_{1,2}^* = \sum_{k=1}^1 t_{1,3-k+1} = t_{1,3} = 5$$

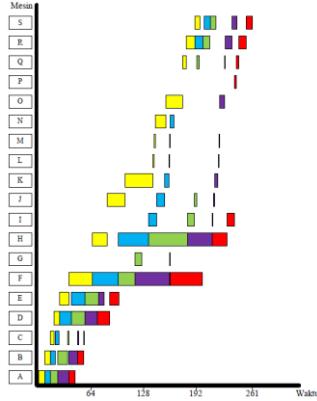
$$t_{1,2}^* = \sum_{k=1}^2 t_{1,3-k+1} = t_{1,3} + t_{1,2}$$

$$= 5 + 3 = 8$$

Set K=2

No	Iterasi	Makespan (Jmn)				
		1	2	3	4	6
1	5 4 3 2 1	294.96				
2	5 3 4 2 1		282.57			
3	4 5 3 2 1			299.42		
4	4 3 5 2 1				282.37	
5	3 4 1 5 2					292.71
6	3 4 5 1 2					278.48
7	3 5 4 1 2					286.48
8	3 5 4 1 2					286.48
9	3 5 1 4 2			284.04		
10	3 5 1 4 2				284.04	
11	3 5 4 2 1					271.52
12	3 5 2 1 4					261.03
13	3 5 2 1 4					261.03
14	3 5 2 1 4					261.03
15	3 5 2 1 4					261.03
16	4 5 3 2 1				298.42	
17	4 5 3 2 1				298.42	
18	5 3 2 1 4				276.7	

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## Palmer

Palmer proposed the calculation of a slope index,  $s_j$ , for each job.

$$s_j = (m-1)t_{j,m} + (m-3)t_{j,m-1} + (m-5)t_{j,m-2} + \dots - (m-3)t_{j,2} - (m-1)t_{j,1}$$

Then a permutation schedule is constructed using the job ordering

$$s_{[1]} \geq s_{[2]} \geq \dots \geq s_{[n]}$$

## Gupta

Gupta thought a transitive job ordering in the form of follows that would produce good schedules. Where

$$s_j = \frac{e_j}{\min\{t_{j1} + t_{j2}, t_{j2} + t_{j3}\}}$$

Where

$$e_j = \begin{cases} 1 & \text{if } t_{jl} < t_{jm} \\ -1 & \text{if } t_{jl} \geq t_{jm} \end{cases}$$

## Gupta

Generalizing from this structure, Gupta proposed that for  $m>3$ , the job index to be calculated is

$$s_j = \frac{e_j}{\min_{1 \leq k \leq m-1} \{t_{jk} + t_{jk+1}\}}$$

Where

$$e_j = \begin{cases} 1 & \text{if } t_{jl} < t_{jm} \\ -1 & \text{if } t_{jl} \geq t_{jm} \end{cases}$$

Ex.

1	2	3	4	5	
t <sub>j1</sub>	6	4	3	9	5
t <sub>j2</sub>	8	1	9	5	6
t <sub>j3</sub>	2	1	5	8	6

CDS: 3-5-4-1-2 M=35

Palmer:  $s_j = (m-1)t_{j3} - (m-1)t_{j1} = 2t_{j3} - 2t_{j1}$   
 $s_1 = -8 \quad s_2 = -6 \quad s_3 = 4 \quad s_4 = -2 \quad s_5 = 2$   
 $\therefore 3-5-4-2-1 \quad M = 37$

Gupta:  $s_1 = -\frac{1}{10} \quad s_2 = -\frac{1}{2} \quad s_3 = \frac{1}{12} \quad s_4 = -\frac{1}{13} \quad s_5 = \frac{1}{11}$   
 $\therefore 5-3-4-1-2 \quad M = 36$

HW.

1	2	3	4	5	
t <sub>j1</sub>	8	11	7	6	9
t <sub>j2</sub>	3	2	5	7	11
t <sub>j3</sub>	6	5	7	13	10

Let  $\sigma = \{1,3\}$

1. Use Ignall-Schrage & McMahon-Burton to solve  $b_1, b_2, b_3, b_4, b_5$  of  $P_{13xxx}^2, P_{31xxx}^2$
2. Use Palmer, Gupta, CDS to solve this problem.

## Referensi

- Introduction to Sequencing and Scheduling. Kenneth R. Baker. Duke University. John Wiley & Sons. 1974.
- Production Scheduling. PPT: Course Material. P.C. Chang. IEM. YZU.

**SELAMAT BELAJAR**