PENELITIAN OPERASIONAL I

(TIN 4109)

Lecture 14

Lecture 14

- **Outline:**
  - Assignment Problem
- **References:**
  - Bazara, Mokhtar S. and Jarvis, John J., *Linear Programming And Network Flows*. John Wiley & Sons, Inc., 1977.
  - Hillier, Frederick and Lieberman, Gerald J., *Introduction to Operations Research*. 7th ed. The McGraw-Hill Companies, Inc., 2001.
  - Taha, Hamdy A., *Operations Research: An Introduction*. 8th Edition. Prentice-Hall, Inc., 2007.

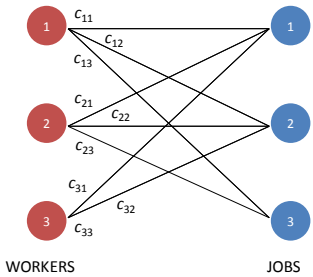
Assignment Problem

- Equal numbers of jobs and workers (or jobs and machines, or …)
- We want to assign the jobs to the workers in a one-to-one manner
- Different job/worker combinations have different costs (or completion times)
- We want to minimize total cost (time)

The Problem....

- Seeks to minimize the total cost assignment of *m* workers to *m* jobs, given that the cost of worker *i* performing job *j* is c_{ij}.
- Assumes all workers are assigned and each job is performed.
- Is a special case of a transportation problem in which all supplies and all demands are equal to 1; hence assignment problems may be solved as linear programs.
- The network representation of an assignment problem with three workers and three jobs is shown on the next slide.

Network Representation



Standard Form LP formulation

$$\min \sum_i \sum_j c_{ij} x_{ij}$$

$$s.t. \quad \sum_j x_{ij} = 1 \quad \text{For each worker } i$$

$$\sum_i x_{ij} = 1 \quad \text{For each job } j$$

$$x_{ij} = 0 \text{ or } 1 \quad \text{For all } i \text{ and } j$$

Note: A modification to the right-hand side of the first constraint set can be made if a worker is permitted to work more than one job.

Assignment Problem – Special Consideration

Special considerations can include :

- A modification to the right-hand side of the first constraint set can be made if a worker is permitted to work more than 1 job.
- number of workers does not equal the number of jobs — add dummy workers/jobs with 0 assignment costs as needed. (note: this may not be necessary if a worker/job is allowed more than one job/worker)
- worker *i* cannot do job *j* — assign $c_{ij} = +M$ *(where +M = some very large number--implying a very large cost!).*

Assignment Problem Example

Four machines are available to process four jobs. The processing time for each machine/job assignment is as follows:

machine \ job	1	2	3	4
A	4	6	5	5
B	7	4	5	6
C	4	7	6	4
D	5	3	4	7

What is the assignment (one job per machine) which will minimize total processing time?

Min.

$4X_{11} + 6X_{12} + 5X_{13} + 5X_{14} + 7X_{21} + 4X_{22} + 5X_{23} + 6X_{24} + 4X_{31} + 7X_{32} + 6X_{33} + 4X_{34} + 5X_{41} + 3X_{42} + 4X_{43} + 7X_{44}$

St.

$X_{11}+X_{12}+X_{13}+X_{14}=1$
$X_{21}+X_{22}+X_{24}+X_{24}=1$
$X_{31}+X_{32}+X_{33}+X_{34}=1$
$X_{41}+X_{42}+X_{43}+X_{44}=1$
$X_{11}+X_{21}+X_{31}+X_{41}=1$
$X_{12}+X_{22}+X_{32}+X_{42}=1$
$X_{13}+X_{23}+X_{33}+X_{43}=1$
$X_{14}+X_{24}+X_{34}+X_{44}=1$

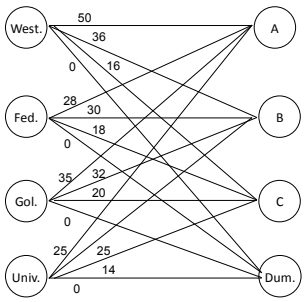
Example: Hungry Owner

A contractor pays his subcontractors a fixed fee plus mileage for work performed. On a given day the contractor is faced with three electrical jobs associated with various projects. Given below are the distances between the subcontractors and the projects.

		Project A	B	C
Subcontractors	Westside	50	36	16
	Federated	28	30	18
	Goliath	35	32	20
	Universal	25	25	14

How should the contractors be assigned to minimize total costs?

Network Representation (note the "dummy" projec



Hungarian Algorithm

- STEP 0 Convert to standard form, with # rows = # columns
- STEP 1 Row reduction: find the smallest cost in each row, and reduce all costs in the row by this amount.
- STEP 2 Column reduction: find the smallest cost in each column, and reduce all costs in the column by this amount.
- STEP 3 Find the minimum number of lines through rows &/or columns necessary to cover all of the zeroes in the cost matrix. If this equals n, then STOP. (Zero assignment)
- STEP 4 Locate the smallest unlined cost. Subtract this cost from all unlined costs, and add to costs at intersections of lines. Return to step 3.

Hungarian Method

- Step 1: Subtract minimum number in each row from all numbers in that row. Since each row has a zero, we would simply generate the same matrix above.
- Step 2: Subtract the minimum number in each column from all numbers in the column. For A it is 25, for B it is 25, for C it is 14, for Dummy it is 0. This yields:

	A	B	C	Dummy
Westside	25	11	2	0
Federated	3	5	4	0
Goliath	10	7	6	0
Universal	0	0	0	0

Hungarian Method (cont'd)

- Step 3: Draw the minimum number of lines to cover all zeroes. Although one can "eyeball" this minimum, use the following algorithm. If a "remaining" row has only one zero, draw a line through the column. If a remaining column has only one zero in it, draw a line through the row.

	A	B	C	Dummy
Westside	25	11	(2)	0
Federated	3	5	4	0
Goliath	10	7	6	0
Universal	~~0~~	~~0~~	~~0~~	~~0~~

- Step 4: The minimum uncovered number is 2 (circled).

Hungarian Method (cont'd)

- Step 5: Subtract 2 from uncovered numbers; add 2 to all numbers covered by two lines. This gives:
- Step 3: Draw the minimum number of lines to cover all zeroes.
- Step 4: The minimum uncovered number is 1 (circled).

	A	B	C	Dummy
Westside	23	9	0	0
Federated	1	3	2	0
Goliath	8	5	4	0
Universal	0	0	0	2

	A	B	C	Dummy
Westside	~~23~~	~~9~~	~~0~~	~~0~~
Federated	(1)	3	2	0
Goliath	8	5	4	0
Universal	~~0~~	~~0~~	~~0~~	~~2~~

Hungarian Method (cont'd)

- Step 5: Subtract 1 from uncovered numbers. Add 1 to numbers covered by two lines. This gives:
- Step 4: The minimum number of lines to cover all 0's is four.

	A	B	C	Dummy
Westside	23	9	0	1
Federated	0	2	1	0
Goliath	7	4	3	0
Universal	0	0	0	3

There is a minimum-cost assignment. The optimal assignment is:

Subcontractor	Project	Distance
Westside	C	16
Federated	A	28
Universal	B	25
Goliath	(unassigned)	
	Total Distance =	69 miles

Exercise 1

- Machinco has four jobs to be completed. Each machine must be assigned to complete one job. The time required to setup each machine for completing each job is shown in the table below. Machinco wants to minimize the total setup time needed to complete the four jobs.

	Time (Hours)			
	Job1	Job2	Job3	Job4
Machine 1	14	5	8	7
Machine 2	2	12	6	5
Machine 3	7	8	3	9
Machine 4	2	4	6	10

Exercise 2

Job/Mach	A	B	C	D
1	8	6	2	4
2	6	7	11	10
3	3	5	7	6
4	5	10	12	9

Lecture 15 – QUIZ 2

- **Materi:**
  - Primal – Dual
  - Dualitas
  - Integer Programming
  - Transportation
  - Assignment

OPEN BUKU (bukan slide)
Catatan: Jangan baru dibaca di kelas.

Lecture 16 – Remedy

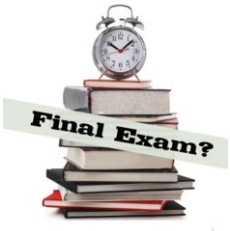
Choose one from two previous test:

- **Quiz 1**
- **Quiz 2**

FINAL EXAM

- **MATERI:**
  - ALL

OPEN NOTE
(A4 - Bolak balik)



SEE YOU