PENELITIAN OPERASIONAL I

(TIN 4109)

Lecture 13

TRANSPORTATION

Lecture 13

- **Outline:**
  - Transportation: optimal solution
- **References:**
  - Bazara, Mokhtar S. and Jarvis, John J., *Linear Programming And Network Flows*. John Wiley & Sons, Inc., 1977.
  - Hillier, Frederick and Lieberman, Gerald J., *Introduction to Operations Research*. 7th ed. The McGraw-Hill Companies, Inc., 2001.
  - Taha, Hamdy A., *Operations Research: An Introduction*. 8th Edition. Prentice-Hall, Inc., 2007.

Exercise

Bazaraa Chapter 10:

Solve the following transportation problem:

		Destination 1	2	3	s_i	
Origin	1	5	4	1	3	
	2	1	7	5	7	c_{ij} matrix
	d_j	2	5	3		

Remember…….

- Those three algorithms require the assumption of

Balanced Transportation Problem

$$\sum_{i=1}^{i=m} s_i = \sum_{j=1}^{j=n} d_j$$

Balancing a Transportation Problem

- If total supply > total demand,
  - adding dummy demand point.
  - Since shipments to the dummy demand point are not real, they are assigned a cost of zero.
- If total supply < total demand (no feasible solution)
  - adding dummy supply point.
  - one or more of the demand will be left unmet.
  - a penalty cost is often associated with unmet demand

Phase 2 : Stepping Stone Method

- Step 1: Select the unoccupied cell with the most positive reduced cost. (For minimization problems select the unoccupied cell with the largest reduced cost.) If none, STOP.

For obtaining reduced costs

Associate a number, u_i, with each row and v_j with each column.

- Step 1: Set $u_1 = 0$.
- Step 2: Calculate the remaining u_i's and v_j's by solving the relationship

  $c_{ij} = u_i + v_j$ for occupied cells.
- Step 3: For unoccupied cells (i,j), the reduced cost $= u_i + v_j - c_{ij}$

Phase 2 : Stepping Stone Method (Cont'd)

- Step 2: For this unoccupied cell generate a stepping stone path by forming a closed loop with this cell and occupied cells by drawing connecting alternating horizontal and vertical lines between them.

  Determine the minimum allocation where a subtraction is to be made along this path.
- Step 3: Add this allocation to all cells where additions are to be made, and subtract this allocation to all cells where subtractions are to be made along the stepping stone path. (Note: An occupied cell on the stepping stone path now becomes 0 (unoccupied).

  If more than one cell becomes 0, make only one unoccupied; make the others occupied with 0's.) GO TO STEP 1.

Example : BBC

- Initial Transportation Tableau

  Since total supply = 100 and total demand = 80, a dummy destination is created with demand of 20 and 0 unit costs.

	Northwood	Westwood	Eastwood	Dummy	Supply
Plant 1	24	30	40	0	50
Plant 2	30	40	42	0	50
Demand	25	45	10	20	

Least Cost

	Northwood	Westwood	Eastwood	Dummy
Plant 1	25 (24)	5 (30)	0 (40)	20 (0)
Plant 2	0 (30)	40 (40)	10 (42)	0 (0)

Phase 2, Iteration 1

- Iteration 1
  1. Set $u_1 = 0$
  2. Since $u_1 + v_j = c_{1j}$ for occupied cells in row 1, then $v_1 = 24$, $v_2 = 30$, $v_4 = 0$.
  3. Since $u_1 + v_2 = c_{i2}$ for occupied cells in column 2, then $u_2 + 30 = 40$, hence $u_2 = 10$.
  4. Since $u_2 + v_j = c_{2j}$ for occupied cells in row 2, then $10 + v_3 = 42$, hence $v_3 = 32$.

	Northwood	Westwood	Eastwood	Dummy	
Plant 1	25 (24)	5 (30)	0 (40)	20 (0)	$u_1 = 0$
Plant 2	0 (30)	40 (40)	10 (42)	0 (0)	$u_2 = 10$
	$v_1 = 24$	$v_2 = 30$	$v_3 = 32$	$v_4 = 0$	

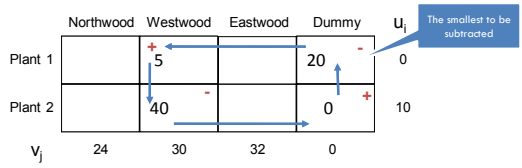
Iteration 1

- Calculate the reduced costs (circled numbers) by $u_i + v_j - c_{ij}$.

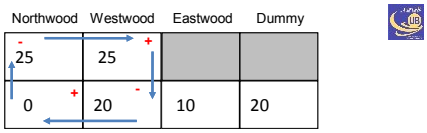
Unoccupied Cell	Reduced Cost
(1,3)	0 + 32 – 40 = -8
(2,1)	24 + 10 – 30 = 4
(2,4)	10 + 0 – 0 = 10

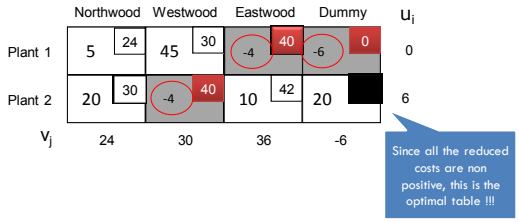
	Northwood	Westwood	Eastwood	Dummy	u_i
Plant 1	25 (24)	5 (30)	-8 (40)	20 (0)	0
Plant 2	+4 (30)	40 (40)	10 (42)	+10 (0)	10
v_j	24	30	32	0	

The most positive unoccupied cell



Iteration 2, Set $u_1 = 0$





Optimal Solution

From	To	Amount	Cost
Plant 1	Northwood	5	120
Plant 1	Westwood	45	1,350
Plant 2	Northwood	20	600
Plant 2	Eastwood	10	420

Total Cost = $2,490

Transportation Simplex Algorithm

STEPS:

1. Check the balance of supply and demand. If it is not balance, balance it using dummy plant (for excess demand) or dummy warehouse
2. Do the starting solution to get basic variable solution (using: heuristic / northwest / VGA method)
3. Check whether the basic variable solution is optimal. The optimality test indicate by for all non basic variable

$$u_i + v_j - c_{ij} \leq 0$$

4. If it is not optimal, conduct the iteration step (stepping stone) to get the optimal solution
   - Determine entering variable & leaving variable
     - Entering variable: the most positive coefficient
     - Leaving variable: satisfying demand and supply quantity; no positive shipments cause by the transfer number of it
   - Construct closing loop

Application in Location Problems

- Seers Inc. telah memiliki 2 *plants* yang melayani permimtaan di 4 kota. Saat ini Seers Inc. sedang mempertimbangkan untuk membuka satu cabang lagi. Alternatif yang dimiliki adalah Atlanta atau Pitsburg. Kapasitas maksimum yang diharapkan pada *plant* yang baru sebesar 330.

  Catatan: kedua alternatif tempat baru tidak membatasi kapasitas.

  TENTUKAN TEMPAT MANA YANG PALING SESUAI UNTUK MENDIRIKAN PLANT BARU.

Data Costs, Demand, dan Supply adalah sbb:

	Boston	Philadel-phia	Galveston	Raleigh	Supply Capacity
Albany	10	15	22	20	250
Little Rock	19	15	10	9	300
Atlanta	21	11	13	6	No Limit
Pitsburg	17	8	18	12	No Limit
Demand	200	100	300	280	

Matrix Table:

SUMBER	TUJUAN				Supply Capacity
	Boston	Philadelphia	Galveston	Raleigh	
Albany	$ 10	$ 15	$ 22	$ 20	250
Little Rock	$ 19	$ 15	$ 10	$ 9	300
Atlanta	$ 21	$ 11	$ 13	$ 6	330
Demand	200	100	300	280	880

SUMBER	TUJUAN				Supply Capacity
	Boston	Philadelphia	Galveston	Raleigh	
Albany	$ 10	$ 15	$ 22	$ 20	250
Little Rock	$ 19	$ 15	$ 10	$ 9	300
Pitsburg	$ 17	$ 8	$ 18	$ 12	330
Demand	200	100	300	280	880

Northwest Corner Method

SUMBER	TUJUAN								Supply Capacity
	Boston		Philadelphia		Galveston		Raleigh		
Albany	200	$ 10	50	$ 15		$ 22		$ 20	**250**
Little Rock		$ 19	50	$ 15	250	$ 10		$ 9	**300**
Atlanta		$ 21		$ 11	50	$ 13	280	$ 6	**330**
Demand	**200**		**100**		**300**		**280**		**880**

Z (alt.1) = $2000 + $750 + $750 + $2500 + $650 + $1680 = **$8330**

SUMBER	TUJUAN								Supply Capacity
	Boston		Philadelphia		Galveston		Raleigh		
Albany	200	$ 10	50	$ 15		$ 22		$ 20	**250**
Little Rock		$ 19	50	$ 15	250	$ 10		$ 9	**300**
Pitsburg		$ 17		$ 8	50	$ 18	280	$ 12	**330**
Demand	**200**		**100**		**300**		**280**		**880**

Z (alt.2) = $2000 + $750 + $750 + $2500 + $900 + $3360 = **$10260**

Final Solution

SUMBER	TUJUAN								Supply Capacity
	Boston		Philadelphia		Galveston		Raleigh		
Albany	200	$ 10	50	$ 15		$ 22		$ 20	**250**
Little Rock		$ 19		$ 15	300	$ 10		$ 9	**300**
Atlanta		$ 21	50	$ 11		$ 13	280	$ 6	**330**
Demand	**200**		**100**		**300**		**280**		**880**

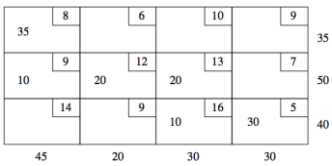
Z (alt.1) = $2000 + $750 + $3000 + $550 + $1680 = **$7980**

SUMBER	TUJUAN								Supply Capacity
	Boston		Philadelphia		Galveston		Raleigh		
Albany	200	$ 10	50	$ 15		$ 22		$ 20	**250**
Little Rock		$ 19		$ 15	300	$ 10		$ 9	**300**
Pitsburg		$ 17	50	$ 8		$ 18	280	$ 12	**330**
Demand	**200**		**100**		**300**		**280**		**880**

Z (alt.2) = $2000 + $750 + $3000 + $400 + $3360 = **$9510**

Latihan Soal:

Consider the following BFS. Let's try to continue on Phase 2.



Latihan Soal:

- Sebuah perusahaan mempunyai 4 lokasi pabrik (pabrik P1, P2, P3 dan P4) dan 3 pusat distribusi (D1, D2 dan D3). pabrik P1, P2 P3 dan P4 perbulannya mampu menghasilkan masing-masing 996 ton; 288 ton; 672 ton dan 444 ton produk A. Sementara pusat distribusi D1, D2 dan D3 kebutuhan perbulannya masing-masing adalah 936 ton, 768 ton dan 696 ton. Biaya transportasi (dalam ribu rupiah per ton) dari setiap pabrik ke masing-masing pusat distribusi diberikan sebagai berikut :

	D1	D2	D3
P1	36	48	60
P2	78	72	90
P3	66	96	54
P4	105	114	84

- Tentukan pengaturan transportasi yang memberikan ongkos yang minimum (solusi optimal) dimana untuk penentuan solusi awal gunakan metode VAM !

Latihan Soal: Televco

Televco produces TV at three plants. Plant 1 can produce 50 pcs per week, plant 2 @100 pcs per week, and plant 3 @50 pcs per week. The profit earned per TV depends on the customer and where the TV was produced (as shown in the Table). Customer 1, 2, and 3 are willing to purchase 80, 90, and 100 respectively. Televco wants to find a shipping & production plan that will maximize profits.

	To ($)		
From	Customer 1	Customer 2	Customer 3
Plant 1	75	60	69
Plant 2	79	73	68
Plant 3	85	76	70

a. Formulate a balanced transportation problem that can be used to maximize Televco's profits
b. Use the Northwest corner method to find a bfs to the problem
c. Use the transportation simplex to find an optimal solution to the problem.

TUGAS

1. Steelco manufactures three types of steel at different plants. The time required to manufacture 1 ton of steel (regardless of type) and the costs at each plant are shown in Table 8. Each week, 100 tons of each type of steel (1, 2, and 3) must be produced. Each plant is open 40 hours per week.
   a. Formulate a balanced transportation problem to minimize the cost of meeting Steelco's weekly requirements.
   b. Find the initial BFS for the problem, and then solve it using Stepping Stone Method

	Cost ($)			Time
Plant	Steel 1	Steel 2	Steel 3	(minutes)
1	60	40	28	20
2	50	30	30	16
3	43	20	20	15

Lecture 14 – Preparation

- **Materi:**
  - Assignment

SEE YOU