PENELITIAN OPERASIONAL I

(TIN 4109)

Lecture 12

TRANSPORTATION

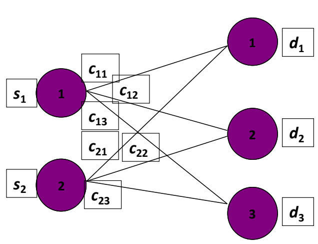
Lecture 12

- **Outline:**
  - Transportation: starting basic feasible solution
- **References:**
  - Frederick Hillier and Gerald J. Lieberman. *Introduction to Operations Research*. 7th ed. The McGraw-Hill Companies, Inc, 2001.
  - Hamdy A. Taha. *Operations Research: An Introduction*. 8th Edition. Prentice-Hall, Inc, 2007.

Transportation

Description

A transportation problem basically deals with the problem, which aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points.



Transportation Problem

- LP Formulation

  The linear programming formulation in terms of the amounts shipped from the origins to the destinations, x_{ij}, can be written as:

$$\text{Min } \sum_i \sum_j c_{ij} x_{ij}$$

$$\text{s.t. } \sum_j x_{ij} \le s_i \text{ for each source } i$$

$$\sum_i x_{ij} >= d_j \text{ for each destination } j$$

$$x_{ij} \ge 0 \text{ for all } i \text{ and } j$$

Transportation Problem

- LP Formulation Special Cases

  The following special-case modifications to the linear programming formulation can be made:

  – Minimum shipping guarantees from i to j:

  $x_{ij} \geq L_{ij}$

  – Maximum route capacity from i to j:

  $x_{ij} \leq L_{ij}$

  – Unacceptable routes:

  delete the variable

Example: BBC

Building Brick Company (BBC) has orders for 80 tons of bricks at three suburban locations as follows: Northwood -- 25 tons, Westwood -- 45 tons, and Eastwood -- 10 tons. BBC has two plants, each of which can produce 50 tons per week.

How should end of week shipments be made to fill the above orders given the following delivery cost per ton:

	Northwood	Westwood	Eastwood
Plant 1	24	30	40
Plant 2	30	40	42

Example: BBC

- LP Formulation

  – Decision Variables Defined

  x_{ij} = amount shipped from plant i to suburb j

  where i = 1 (Plant 1) and 2 (Plant 2)

  j = 1 (Northwood), 2 (Westwood), and 3 (Eastwood)

Example: BBC

- LP Formulation

  – Objective Function

  Minimize total shipping cost per week:

  Min $24x_{11} + 30x_{12} + 40x_{13} + 30x_{21} + 40x_{22} + 42x_{23}$

  – Constraints

  s.t. $x_{11} + x_{12} + x_{13} \leq 50$ (Plant 1 capacity)

  $x_{21} + x_{22} + x_{23} \leq 50$ (Plant 2 capacity)

  $x_{11} + x_{21} >= 25$ (Northwood demand)

  $x_{12} + x_{22} >= 45$ (Westwood demand)

  $x_{13} + x_{23} >= 10$ (Eastwood demand)

  all $x_{ij} \geq 0$ (Non-negativity)

Exercise….

Powerco has three electric power plants that supply the electric needs of four cities.

- The associated supply of each plant and demand of each city is given in the table as follows:
- The cost of sending 1 million kwh of electricity from a plant to a city depends on the distance the electricity must travel.

From	*To*				
	City 1	*City 2*	*City 3*	*City 4*	*Supply (Million kwh)*
Plant 1	$8	$6	$10	$9	35
Plant 2	$9	$12	$13	$7	50
Plant 3	$14	$9	$16	$5	40
Demand (Million kwh)	45	20	30	30	

Solution

1. Decision Variable:

   Since we have to determine how much electricity is sent from each plant to each city;

   X_{ij} = Amount of electricity produced at plant i and sent to city j

   X_{14} = Amount of electricity produced at plant 1 and sent to city 4

2. Objective Function:

   Since we want to minimize the total cost of shipping from plants to cities;

   Minimize $Z = 8X_{11}+6X_{12}+10X_{13}+9X_{14}+9X_{21}+12X_{22}+13X_{23}+7X_{24}+14X_{31}+9X_{32}+16X_{33}+5X_{34}$

Solution

3. Supply Constraints

$X_{11}+X_{12}+X_{13}+X_{14} <= 35$
$X_{21}+X_{22}+X_{23}+X_{24} <= 50$
$X_{31}+X_{32}+X_{33}+X_{34} <= 40$

Since each supply point has a limited production capacity;

4. Demand Constraints

$X_{11}+X_{21}+X_{31} >= 45$
$X_{12}+X_{22}+X_{32} >= 20$
$X_{13}+X_{23}+X_{33} >= 30$
$X_{14}+X_{24}+X_{34} >= 30$

Since each supply point has a limited production capacity;

5. Sign Constraints

$X_{ij} >= 0$ (i= 1,2,3; j= 1,2,3,4)

LP Formulation of Powerco's Problem

Min $Z = 8X_{11}+6X_{12}+10X_{13}+9X_{14}+9X_{21}+12X_{22}+13X_{23}+7X_{24}$ $+14X_{31}+9X_{32}+16X_{33}+5X_{34}$

S.T.: $X_{11}+X_{12}+X_{13}+X_{14} <= 35$ **(Supply Constraints)**
$X_{21}+X_{22}+X_{23}+X_{24} <= 50$
$X_{31}+X_{32}+X_{33}+X_{34} <= 40$
$X_{11}+X_{21}+X_{31} >= 45$ **(Demand Constraints)**
$X_{12}+X_{22}+X_{32} >= 20$
$X_{13}+X_{23}+X_{33} >= 30$
$X_{14}+X_{24}+X_{34} >= 30$
$X_{ij} >= 0$ (i= 1,2,3; j= 1,2,3,4)

Balanced Transportation Problem

If Total supply equals to total demand, the problem is said to be a balanced transportation problem:

$$\sum_{i=1}^{i=m} s_i = \sum_{j=1}^{j=n} d_j$$

Finding Basic Feasible Solution for Transportation Problem

Unlike other Linear Programming problems,

a *balanced* TP with m supply points and n demand points is easier to solve, although it has m + n equality constraints.

The reason for that is, if a set of decision variables (x_{ij}'s) satisfy all but one constraint, the values for x_{ij}'s will satisfy that remaining constraint automatically.

Methods to find the BFS for a balanced Transportation Problem

There are three basic methods:

1. North West Corner Method
2. Minimum Cost Method
3. Vogel's Method

1. Northwest Corner Method

To find the bfs by the NWC method:

Begin in the upper left (northwest) corner of the transportation tableau and set x_{11} as large as possible (here the limitations for setting x_{11} to a larger number, will be the demand of demand point 1 and the supply of supply point 1. Your x_{11} value can not be greater than minimum of this 2 values).

According to the explanations in the previous slide we can set $x_{11}=3$ (meaning demand of demand point 1 is satisfied by supply point 1).

				5
				6
				2
3	5	2	3	

3				2
				6
				2
X	5	2	3	

After we check the east and south cells, we saw that we can go east (meaning supply point 1 still has capacity to fulfill some demand).

3	2			X
				6
				2
X	3	2	3	

3	2			X
	3			3
				2
X	X	2	3	

After applying the same procedure, we saw that we can go south this time (meaning demand point 2 needs more supply by supply point 2).

3	2			X
	3	2		1
				2
X	X	X	3	

3	2			X
	3	2	1	X
				2
X	X	X	2	

Finally, we will have the following bfs, which is: $x_{11}=3$, $x_{12}=2$, $x_{22}=3$, $x_{23}=2$, $x_{24}=1$, $x_{34}=2$

3	2			X
	3	2	1	X
			2	X
X	X	X	X	

2. Minimum Cost Method

The Northwest Corner Method dos not utilize shipping costs.

It can yield an initial BFS easily but the total shipping cost may be very high.

The minimum cost method uses shipping costs in order come up with a BFS that has a lower cost.

2. Minimum Cost Method (Cont'd)

The Steps:

1. First, We look for the cell with the minimum cost of shipping in the overall transportation tableau.
2. We should cross out row i and column j and reduce the supply or demand of the noncrossed-out row or column by the value of Xij.
3. Then we will choose the cell with the minimum cost of shipping from the cells that do not lie in a crossed-out row or column and we will repeat the procedure.

An example for Minimum Cost Method
Step 1: Select the cell with minimum cost.

2	3	5	6	5
2	1	3	5	10
3	8	4	6	15
12	8	4	6	

Step 2: Cross-out column 2

2	3	5	6	5
2	1 8	3	5	2
3	8	4	6	15
12	X	4	6	

Step 3: Find the new cell with minimum shipping cost and cross-out row 2

2	3	5	6	5
2 2	1 8	3	5	X
3	8	4	6	15
10	X	4	6	

Step 4: Find the new cell with minimum shipping cost and cross-out row 1

2 5	3	5	6	X
2 2	1 8	3	5	X
3	8	4	6	15
5	X	4	6	

Step 5: Find the new cell with minimum shipping cost and cross-out column 1

2 5	3	5	6	X
2 2	1 8	3	5	X
3 5	8	4	6	10
X	X	4	6	

Step 6: Find the new cell with minimum shipping cost and cross-out column 3

2 5	3	5	6	X
2 2	1 8	3	5	X
3 5	8	4 4	6	6
X	X	X	6	

Step 7: Finally assign 6 to last cell. The bfs is found as: $X_{11}=5$, $X_{21}=2$, $X_{22}=8$, $X_{31}=5$, $X_{33}=4$ and $X_{34}=6$

2 / 5	3	5	6	X
2 / 2	1 / 8	3	5	X
3 / 5	8	4 / 4	6 / 6	X
X	X	X	X	

3. Vogel's Method

1. Begin with computing each row and column a penalty. The penalty will be equal to the difference between the two smallest shipping costs in the row or column.
2. Identify the row or column with the largest penalty.
3. Find the first basic variable which has the smallest shipping cost in that row or column.
4. Then assign the highest possible value to that variable, and cross-out the row or column as in the previous methods. Compute new penalties and use the same procedure.

An example for Vogel's Method
Step 1: Compute the penalties.

				Supply	Row Penalty
	6	7	8	10	7-6=1
	15	80	78	15	78-15=63
Demand	15	5	5		
Column Penalty	15-6=9	80-7=73	78-8=70		

Step 2: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6	7 / 5	8	5	8-6=2
	15	80	78	15	78-15=63
Demand	15	X	5		
Column Penalty	15-6=9	_	78-8=70		

Step 3: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6	7 / 5	8 / 5	0	_
	15	80	78	15	_
Demand	15	X	X		
Column Penalty	15-6=9	_	_		

Step 4: Identify the largest penalty and assign the highest possible value to the variable.

				Supply	Row Penalty
	6 / 0	7 / 5	8 / 5	X	_
	15	80	78	15	_
Demand	15	X	X		
Column Penalty	_	_	_		

Step 5: Finally the bfs is found as X_{11}=0, X_{12}=5, X_{13}=5, and X_{21}=15

				Supply	Row Penalty
	6 / 0	7 / 5	8 / 5	X	_
	15 / 15	80	78	X	_
Demand	X	X	X		
Column Penalty	_	_	_		

Exercise

Bazaraa Chapter 10:

Solve the following transportation problem:

Destination

		1	2	3	s_i	
Origin	1	5	4	1	3	
	2	1	7	5	7	c_{ij} matrix
	d_j	2	5	3		

Exercise

Bazaraa Chapter 10:

Solve the following transportation problem:

	1	2	3	4	s_i	
1	7	2	-1	0	10	
2	4	3	2	3	30	c_{ij} matrix
3	2	1	3	4	25	
d_j	10	15	25	15		

Lecture 13 – Preparation

- **Materi:**
  - Transportasi: Optimum Solution

SEE YOU