PENELITIAN OPERASIONAL I

(TIN 4109)

Lecture 7

LINEAR PROGRAMMING

Lecture 7

- **Outline:**
  - Duality

- **References:**
  - Frederick Hillier and Gerald J. Lieberman. *Introduction to Operations Research*. 7th ed. The McGraw-Hill Companies, Inc, 2001.
  - Hamdy A. Taha. *Operations Research: An Introduction*. 8th Edition. Prentice-Hall, Inc, 2007.

DUALITAS

Linear Programming

- LP dalam bentuk standard

$$\text{Maximize or minimize } z = \sum_{j=1}^{n} c_j x_j$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, i = 1, 2, \ldots, m$$

$$x_j \geq 0, j = 1, 2, \ldots, n$$

Linear Programming

	Primal variables						
	x_1	x_2	$\cdots$	x_j	$\cdots$	x_n	
Dual variables	c_1	c_2	$\cdots$	c_j	$\cdots$	c_n	Right-hand side
y_1	a_{11}	a_{12}	$\cdots$	a_{1j}	$\cdots$	a_{1n}	b_1
y_2	a_{21}	a_{22}	$\cdots$	a_{2j}	$\cdots$	a_{2n}	b_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	
y_m	a_{m1}	a_{m2}	$\cdots$	a_{mj}	$\cdots$	a_{mn}	b_m
				↑ jth dual constraint			↑ Dual objective coefficients

Linear Programming

Kondisi keoptimalan:

$$z_0 = y\,b = y_1b_1 + y_2b_2 + \dots + y_mb_m \qquad (1)$$

$$z_j = y\,a_j = y_1a_{1j} + y_2a_{2j} + \dots + y_ma_{mj} \qquad (2)$$

sehingga persoalan LP dapat diintepretasikan sebagai berikut:

cari $y_1, y_2, \dots, y_m$ sedemikian hingga (1) dan (2) terpenuhi

Linear Programming

Dapat dilakukan dengan menyelesaikan LP sebagai berikut:

Min $z_0 = y_1b_1 + y_2b_2 + \dots + y_mb_m$

subject to

$$y_1a_{1j} + y_2a_{2j} + \dots + y_ma_{mj} \geq c_j$$

$$y_1\ ,\ y_2\ ,\ \ y_m \geq 0$$

Maka diperoleh problem LP yang baru yang disebut DUAL dari problem semula atau disingkat PROBLEM DUAL

Primal and Dual

Primal Problem	Dual Problem
Max $Z = \sum_{j=1}^{n} c_j x_j$,	Min $W = \sum_{i=1}^{m} b_i y_i$,
s.t. $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$, for $i = 1, 2, \dots, m$.	s.t. $\sum_{i=1}^{m} a_{ij} y_i \geq c_j$, for $j = 1, 2, \dots, n$.
$x_j \geq 0$, for $j = 1, 2, \dots, n$.	$y_i \geq 0$, for $i = 1, 2, \dots, m$.

The dual problem uses exactly the same parameters as the primal problem, but in different location.

Primal Dual dalam Matriks

Primal Problem	Dual Problem
Maximize $Z = cx$,	**Minimize** $W = yb$,
subject to	**subject to**
$Ax \leq b$	$yA \geq c$
$x \geq 0.$	$y \geq 0.$

Where c and $y = [y_1, y_2, \dots, y_m]$ are row vectors but b and x are column vectors.

Contoh: Primal – Dual

Primal Problem in Algebraic Form	Dual Problem in Algebraic Form
Max $Z = 3x_1 + 5x_2$,	**Min** $W = 4y_1 + 12y_2 + 18y_3$,
s.t. $x_1 \leq 4$	**s.t.** $y_1 + 3y_3 \geq 3$
$2x_2 \leq 12$	$2y_2 + 2y_3 \geq 5$
$3x_1 + 2x_2 \leq 18$	$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0$
$x_1 \geq 0, x_2 \geq 0$	

Programa Dual

Hubungan antara PRIMAL dan DUAL adalah sebagai berikut :

PRIMAL		DUAL
RHS		Fungsi Tujuan
MAX	⟺	MIN
Constrain		Variable

Programa Dual

PRIMAL

DUAL	x_1	x_2	$\dots$	x_n	RHS
y_1	a_{11}	a_{12}	$\dots$	a_{1n}	$\le b_1$
y_2	a_{21}	a_{22}	$\dots$	a_{2n}	$\le b_2$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
y_m	a_{m1}	a_{m2}		a_{mn}	$\le b_m$
	$\ge$	$\ge$		$\ge$	
	c_1	c_2	$\dots$	c_n	

Koefisien Fungsi Objektif (Minimisasi)

Koefisien Fungsi Objektif (Maksimisasi)

Contoh Programa Dual

PRIMAL : Max $3x_1 + 5x_2$

s.t.

$x_1 \le 4$

$2x_2 \le 12$

$3x_1 + 2x_2 \le 18$

$x_1, x_2 \ge 0$

DUAL : Min $4y_1 + 12y_2 + 18y_3$

s.t.

$y_1 + 3y_3 \ge 3$

$2y_2 + 2y_3 \ge 5$

$y_1, y_2, y_3 \ge 0$

DUAL dari DUAL adalah PRIMAL

Primal of Diet problem

min $w = 50y_1 + 20y_2 + 30y_3 + 80y_4$

s.t.
$400y_1 + 200y_2 + 150y_3 + 500y_4 \ge 500$ (Calorie constraint)
$3y_1 + 2y_2 \ge 6$ (Chocolate constraint)
$2y_1 + 2y_2 + 4y_3 + 4y_4 \ge 10$ (Sugar constraint)
$2y_1 + 4y_2 + y_3 + 5y_4 \ge 8$ (Fat constraint)
$y_1, y_2, y_3, y_4 \ge 0$

where

y_1 = number of brownies eaten daily
y_2 = number of scoops of chocolate ice cream eaten daily
y_3 = bottles of soda drunk daily
y_4 = pieces of pineapple cheesecake eaten daily

Diet Problem – Dual

max $z = 500x_1 + 6x_2 + 10x_3 + 8x_4$

s.t.
$400x_1 + 3x_2 + 2x_3 + 2x_4 \le 50$
$200x_1 + 2x_2 + 2x_3 + 4x_4 \le 20$
$150x_1 + 4x_3 + x_4 \le 30$
$500x_1 + 4x_3 + 5x_4 \le 80$
$x_1, x_2, x_3, x_4 \ge 0$

PRIMAL – DUAL

Secara umum hubungan antara DUAL dan PRIMAL dapat digambarkan seperti pada tabel di bawah ini

	MINIMASI		MAKSIMASI	
Variable	$\ge$	$\Leftrightarrow$	$\le$	Constraint
Variable	$\le$	$\Leftrightarrow$	$\ge$	Constraint
Variable	Unrestricted	$\Leftrightarrow$	$=$	Constraint
Constraint	$\ge$	$\Leftrightarrow$	$\ge$	Variable
Constraint	$\le$	$\Leftrightarrow$	$\le$	Variable
Constraint	$=$	$\Leftrightarrow$	Unrestricted	Variable

Contoh

PRIMAL : Max $8x_1 + 3x_2$

s.t.

$x_1 - 6x_2 \ge 4$

$5x_1 + 7x_2 = -4$

$x_1 \le 0$

$x_2 \ge 0$

DUAL : Min $4w_1 - 4w_2$

s.t.

$w_1 + 5w_2 \le 8$

$-6w_1 + 7w_2 \ge 3$

$w_1 \le 0$

w_2 unrestricted

Contoh 2

Primal: Max. $z = 3x_1 + 2x_2$ (Obj. Func.)

subject to

$2x_1 + x_2 \le 100$ (Finishing constraint)

$x_1 + x_2 \le 80$ (Carpentry constraint)

$x_1 \le 40$ (Bound on soldiers)

$x_1, x_2 \ge 0$

Optimal Solution: $z = 180$, $x_1 = 20$, $x_2 = 60$

Selesaikan pendekatan Dual-nya.

Lecture 8 – Preparation

- **Materi:**
  - Duality
  - Analisa Sensitivitas

SEE YOU