PENELITIAN OPERASIONAL I

(TIN 4109)

Lecture 2

LINEAR PROGRAMMING

Lecture 2

- **Outline:**
  - Introduction to Linear Programming
  - Graphic Method

- **References:**
  - Frederick Hillier and Gerald J. Lieberman. *Introduction to Operations Research*. 7th ed. The McGraw-Hill Companies, Inc, 2001.
  - Hamdy A. Taha. *Operations Research: An Introduction*. 8th Edition. Prentice-Hall, Inc, 2007.
  - Sankaranarayanan, Sriram. Lecture Note. University of Colorado Boulder, 2013.

Linear Programming: Definition and Example

- Allocating limited resources among competing activities in a best possible (i.e., optimal) way.
  - Selecting the **level of** certain **activities** that **compete** for **scarce resources** that are necessary.
  - How much each **resouce** will be **consumed by** each **activity.**
- Uses a mathematical model to describe the problem of concern.
  - **Linear**: all mathematical functions in the model must be linear functions.
  - **Programming**: planning.
- Planning activities to obtain an optimal result, i.e., a result that reaches the specified goal best (according to mathematical model) among all feasible alternatives.

Linear Programming: Definition and Example

Decision Variables

x_1, x_2, x_3, x_4

Objective Function

$$\text{maximize} \quad 2x_1 + 3x_2 - x_3 + x_4$$

$$\begin{array}{lccccl} \text{subject to} & x_1 & -x_2 & & & \leq 10 \\ & 2x_1 & +x_2 & -x_3 & & \geq -5 \\ & & -x_2 & & +x_4 & = 4 \end{array}$$

Constraints

Linear Programming: Definition and Example

- Basic components:
  1. Decision **variables** that we seek to determine.
  2. **Objective** (goal) that we need to optimize (maximize or minimize).
  3. **Constraint** that the solution must satisfy.

- Steps:
  1. Proper definition of the decision variables
  2. Constructing the objective function and constraints
  3. Solving the problem

Linear Programming: Definition and Example

- **Example of LP problem:**

– ***Perusahaan kaca WYNDOR memproduksi kaca (jendela dan pintu) dengan kualitas tinggi. Perusahaan ini mempunyai tiga departemen. Departemen 1 membuat rangka alumunium dan perkakas logam, Departemen 2 membuat rangka kayu, dan Departemen 3 membuat kaca dan merakit sebuah produk. Akibat penurunan pendapatan, pihak atasan memutuskan untuk mengubah lini produk perusahaan. Produk yang tidak mendatangkan keuntungan akan dihentikan dan perusahaan akan menentukan kapasitas produk untuk membuat dua produk baru yang dinilai mempunyai potensi pasar tinggi.***

Produk 1: pintu kaca dengan rangka aluminium berukuran 8 kaki

Produk 2: rangka rangkap jendela dari kayu berukuran 4 x 6 kaki

Produk 1 membutuhkan proses di Departemen 1 dan 3. Produk 2 diproses pada Departemen 2 dan 3. Perusahaan memiliki data sebagai berikut:

Departemen	**Waktu Prod / Batch (Jam)**		**Waktu Prod yg tersedia / minggu (Jam)**
	Produk 1	Produk 2	
1	1	0	4
2	0	2	12
3	3	2	18
Keuntungan/batch	\$3000	\$5000	

- Define the variables

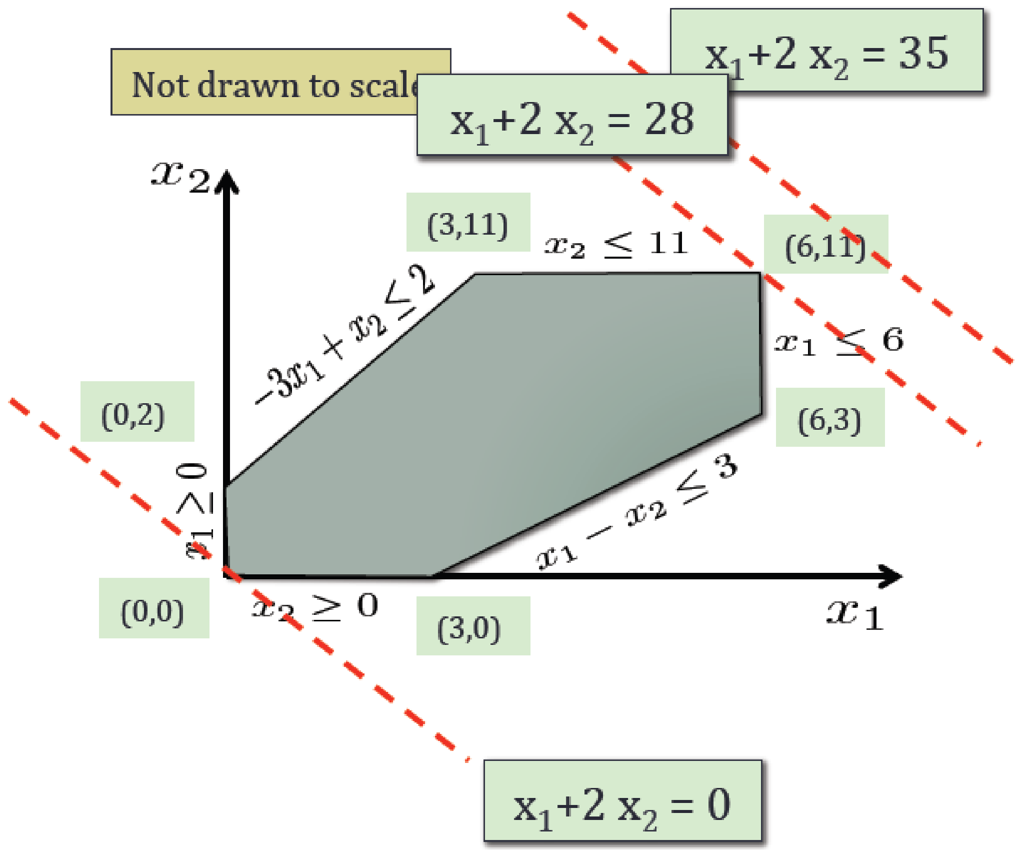
- Construct the objective function and constraints

Linear Programming: Graphic Method

- Example of LP

$$
\begin{array}{llllll}
\text{max.} & x_1 & +2x_2 & & \\
\text{s.t.} & -3x_1 & +x_2 & \leq & 2 \\
 & & +x_2 & \leq & 11 \\
 & x_1 & -x_2 & \leq & 3 \\
 & x_1 & & \leq & 6 \\
 & x_1, & x_2 & \geq & 0
\end{array}
$$

Solution: $x_1 = 6, x_2 = 11$
Optimal Objective Value: 28



Linear Programming: Graphic Method

- Selesaikan permasalahan contoh soal perusahaan WYNDOR dengan menggunakan metode grafik.

Linear Programming: Visualizing

- General Form of LP:

Objective Function

$\{\leq, \geq, =\}$

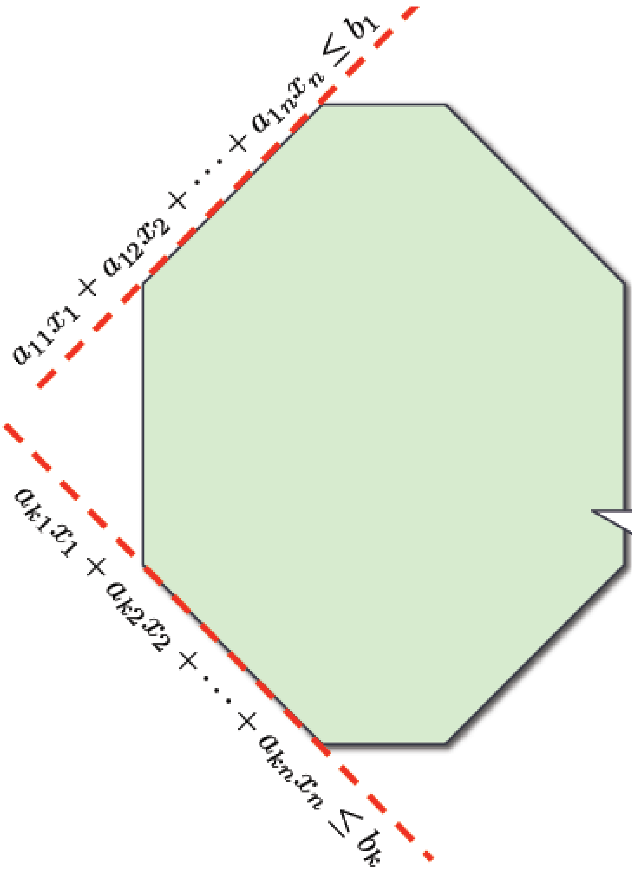
$$\begin{array}{lllllll} \max & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & \\ \text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 \\ & & & \ddots & & \vdots & \\ & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m \end{array}$$

Constraints

Linear Programming: Visualizing

- Feasible Region:

$$
\begin{array}{lccccccc}
\text{max} & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & \\
\text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 \\
 & & & \ddots & & \vdots & \\
 & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m
\end{array}
$$

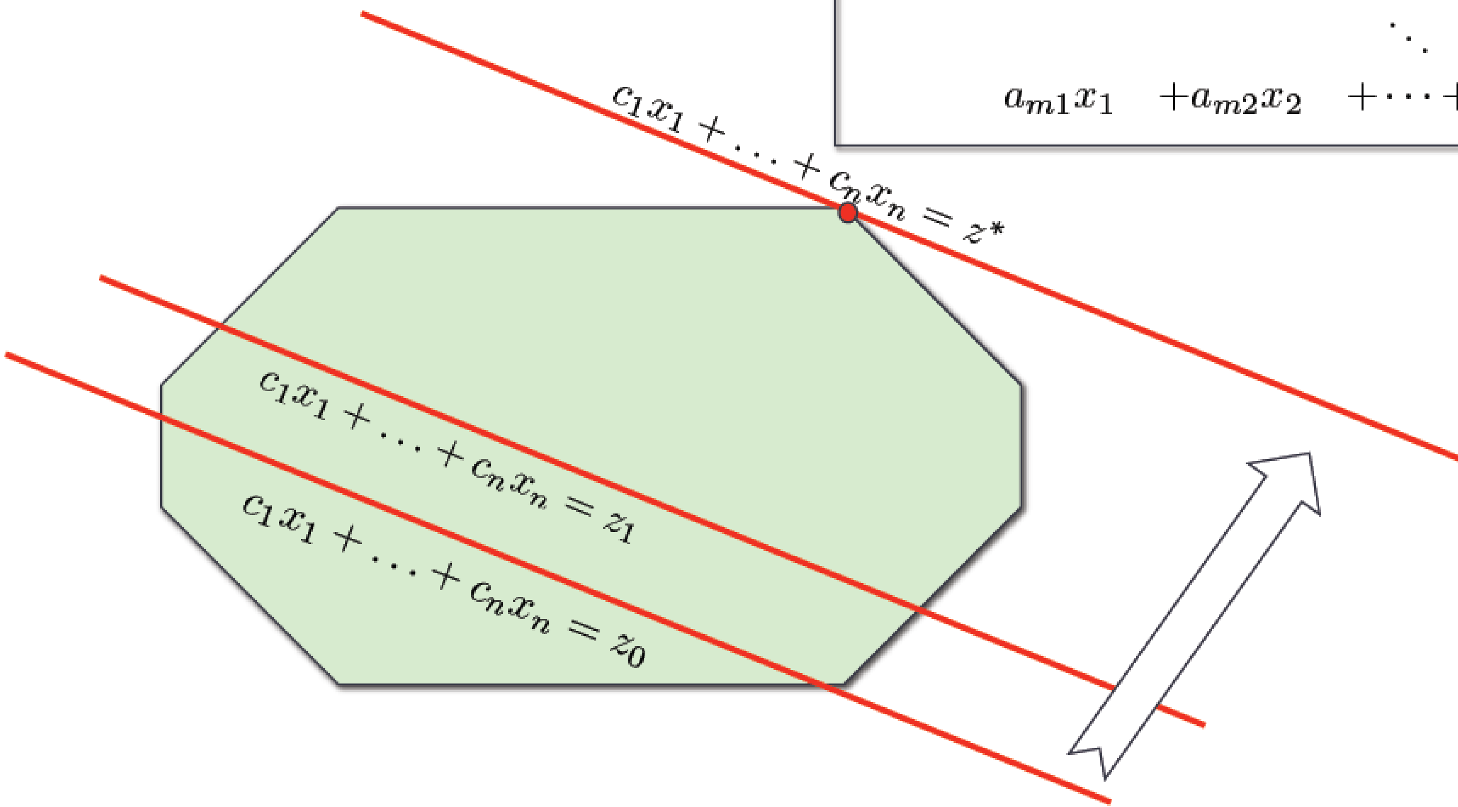


Feasible Region: Polyhedron
(n dimensional)

Linear Programming: Visualizing

- Optimization

$$
\begin{array}{lccccccc}
\text{max} & c_1x_1 & +c_2x_2 & +\cdots+ & c_nx_n & & \\
\text{s.t.} & a_{11}x_1 & +a_{12}x_2 & +\cdots+ & a_{1n}x_n & \leq & b_1 \\
 & & & \ddots & & \vdots & \\
 & a_{m1}x_1 & +a_{m2}x_2 & +\cdots+ & a_{mn}x_n & \leq & b_m
\end{array}
$$

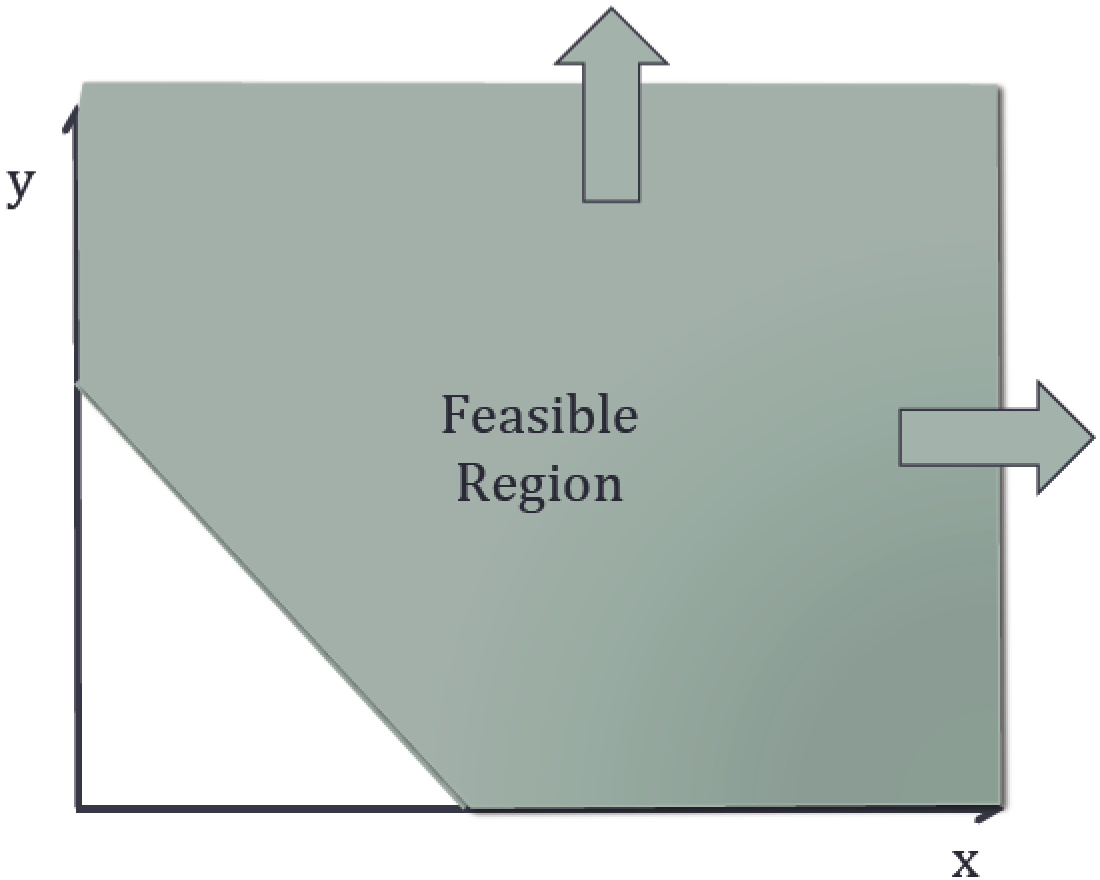


Linear Programming: Visualizing

- Solving Linear Problems:
  - Outcome #1: Optimal solution(s) exists
  - Outcome #2: Objective function is unbounded
  - Outcome #3: Feasible region is empty

Linear Programming: Visualizing

- Unbounded Problem (Example):



$$
\begin{array}{llll}
\max & x & & \\
\text{s.t.} & x & \geq & 0 \\
 & x + y & \geq & 1 \\
 & y & \geq & 0
\end{array}
$$

Linear Programming: Visualizing

- Infeasible Problem:
  - Issue: Constraints contradict each other.

$$
\begin{array}{rllll}
\max & x & & & \\
\text{s.t.} & x & & \geq & 0 \\
& \boxed{x} & \boxed{+y} & \boxed{\geq} & \boxed{1} \\
& & y & \geq 0 & \\
& \boxed{x} & \boxed{+y} & \boxed{\leq} & \boxed{\frac{1}{2}}
\end{array}
$$

Linear Programming: Visualizing

- Solving Linear Problems:
  1. Find which of three cases are applicable.
     - Infeasible?
     - Unbounded?
     - Feasible + Bounded = Optimal?
  2. If Optimal, find optimal solution.
     - Note multiple optimal solutions possible

Linear Programming: Properties

- Proportionality
  - Contribution of each decision variable in both objective function and the constraints to be directly proportional to the value of the variable.
- Additivity
  - Total contribution of all the variables in the objective function and in the contraints to be the direct sum of the individual contributions of each variable.
- Certainty
  - All the objective and contraints coefficients of linear programming model are deterministic.
- Divisibility
  - Desicion variables in a linear programming model are allowed to have any values, including *noninteger* values, that satisfy the functional and nonnegativity contraints.

Linear Programming: Algorithms

- Solving systems of Linear Ineaqualities
  - Early work by Fourier (Fourier-Motzkin Elimination Algorithm).
  - Linear Arithmetic.

- World War II: Optimal allocation of resources.
  - Advent of electronic / mechanical calculating machines.
  - L.V. Kantorovich in USSR (1940) and G.B. Dantzig et al. In the USA (1947).

Linear Programming: Algorithms

- Simplex.
- Ellipsoidal Methods.
- Interior Point Methods.

LP: Exercises

- **Buat formulasi LP dari permasalahan berikut:**

  NORI & LEETS CO., salah satu produsen baja utama di dunia, bertempat di kota Steeltown dan satu-satunya perusahaan yang mempunyai banyak pekerja. Perusahaan memiliki dua sumber polusi utama: Blast Furnaces (BF) dan Open-Heart Furnaces (OF). Perusahaan ingin memperbaiki kualitas lapisan udara di kota Steeltown dengan mengurangi polutan yang mereka timbulkan. Adapun data yang dimiliki adalah sebagai berikut:

Polutan	Laju Pengurangan pada Emisi Tahunan yg diperlukan (Juta Pound)
Zat khusus	60
Sulfur oksida	150
Hidrokarbon	125

Total biaya yang dapat digunakan utk pengurangan (Juta dolar)		
Metode Pengurangan	**BF**	**OF**
(1)	8	10
(2)	7	6
(3)	11	9

Reduksi laju emisi (Jutaan pound) dari metode pengurangan yg dilakukan Nori&Leets Co.						
Polutan	**Tambah tinggi cerobong asap (1)**		**Menggunakan alat penyaring di cerobong (2)**		**Menggunakan bahan bakar yg lebih baik (3)**	
	BF	**OF**	**BF**	**OF**	**BF**	**OF**
Zat khusus	12	9	25	20	17	13
Sulfur oksida	35	42	18	31	56	49
Hidrokarbon	37	53	28	24	29	20

LP: Exercises

- **Selesaikan dengan menggunakan metode grafik persoalan berikut:**

$$Maximize\ Z = 2x_1 + x_2$$

$Subject\ to$:

$$-x_1 + 2x_2 \leq 15$$
$$x_1 + 2x_2 \leq 12$$
$$5x_1 + 3x_2 \leq 45$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Lecture 3 - Preparation

- **Read and Practice:**
  - Hamdy A. Taha. *Operations Research: An Introduction*. 8th Edition. Prentice-Hall, Inc, 2007. Page: 27-42 (Examples 2.3.1 to 2.3.3).

SEE YOU